Math Day 2020
at Murray State University
Upper Level Examination

- Do not open this exam until you are told to do so.
- Clearly fill in your NAME and STUDENT NUMBER on the bubble sheet. Your student number is located on the card your teacher gave you.
- You have 60 minutes to complete this exam.
- You may not use a calculator, phone, notes, book, or other aid. Any attempt to do so will result in disqualification.
- The exam will be scored as follows:

  +1 point for a correct answer
  $-\frac{1}{4}$ point for an incorrect answer
  0 points for a blank answer

- Clearly select one answer on the bubble sheet for each question. If more than one answer is selected, the answer will be marked as incorrect.

GOOD LUCK!

1. Suppose at 3:00 pm, Pat enters the highway at point P headed east traveling at 50 mph. 30 minutes later, Tom enters the same highway at a point 10 miles west of point P headed east traveling at 70 mph. At what time will Tom catch Pat?

   (a) 4:45 pm
   (b) 5:00 pm
   (c) 5:15 pm
   (d) 5:30 pm
   (e) None of the above

2. Let $F(t) = \int_0^t [17 - (x-3)^2]dx$ be defined on $D = [0, \infty)$. Find where F achieves its absolute maximum value on D.

   (a) $x = 3$
   (b) $x = \sqrt{17}$
   (c) $x = 3 + \sqrt{17}$
   (d) $x = 3 - \sqrt{17}$
   (e) There is not an absolute maximum value.

3. Suppose the following statement is false:

   They like football, but they neither like baseball nor hockey.

   Which of the following is always a true statement?

   (a) If they don't like football, then they either like baseball or hockey.
   (b) If they like football, then they either like baseball or hockey.
   (c) If they don't like football, then they neither like baseball nor hockey.
   (d) If they like football, then they neither like baseball nor hockey.
   (e) They don't like football, but they either like baseball or hockey.

4. Suppose that a ten-sided die has possible outcomes 1-10, each with equal probability. If the die is rolled 3 times, find the probability that you won't get a 10 until the 3rd roll.

   (a) .008
   (b) .009
   (c) .08
   (d) .09
   (e) None of the above

5. Suppose that

$$x = \frac{2}{2 + \frac{2}{2+\frac{2}{2+\ldots}}}.$$

   Then

   (a) $x = -1 + \sqrt{2}$.
   (b) $x = -1 + \sqrt{3}$.
   (c) $x = -2 + \sqrt{5}$.
   (d) $x = -2 + \sqrt{6}$.
   (e) $x = -2 + \sqrt{7}$.

6. Let $x \neq 0$ satisfy $-2 \leq x \leq 2$. Calculate $\tan\left(\cos^{-1}\left(\frac{x}{2}\right)\right)$.

   (a) $\frac{x}{\sqrt{1-x^2}}$.
   (b) $\frac{\sqrt{1-x^2}}{x}$.
   (c) $\frac{x}{\sqrt{4-x^2}}$.
   (d) $\frac{\sqrt{4-x^2}}{x}$.
   (e) None of the above.

7. Suppose IQ scores are normally distributed with a mean of 100 and standard deviation of 15. If two people are chosen randomly and independently, what is the approximate probability that both will have an IQ above 130?

   (a) .000625

   (b) .0025

   (c) .025

   (d) .05

   (e) None of the above.

8. Simplify the following expression when $x \neq 1$:

$$\frac{x^4 - 14x^3 + 60x^2 - 82x + 35}{x - 1}.$$

   (a) $x^3 - 14x^2 + 60x - 82 + \dfrac{35}{x}$

   (b) $x^4 - 14x^3 + 60x^2 - 82 + 34$

   (c) $x^3 + 13x^2 - 47x + 35$

   (d) $x^3 - 13x^2 + 47x - 35$

   (e) None of the above

9. Suppose a car's wheels are moving at a rate of 2000 revolutions per minute. If the radius of all 4 wheels (including the tires) is 1.5 feet, find the linear speed of the car.

   (a) $3000\dfrac{feet}{minute}$

   (b) $\dfrac{4000}{3}\dfrac{feet}{minute}$

   (c) $\dfrac{8000\pi}{3}\dfrac{feet}{minute}$

   (d) $4000\pi\dfrac{feet}{minute}$

   (e) None of the above

10. Calculate the following sum:

$$1 + 4 + 7 + 10 + 13 + 16 + ... + 298$$

(a) 13950

(b) 14950

(c) 15950

(d) 16950

(e) None of the above

11. Find where the tangent line to $f(x) = 2^x$ at $x = 1$ intersects the x-axis.

(a) $x = \dfrac{1}{\ln(2)}$

(b) $x = 2\ln(2)$

(c) $x = \dfrac{\ln(2) - 1}{\ln(2)}$

(d) $x = \dfrac{\ln(2)}{\ln(2) - 1}$

(e) None of the above

12. Simplify the following expression:

$$\sin^2(\theta) + \cos^2(\theta)(2\tan(\theta) + 1)$$

(a) $2\tan(\theta) + 1$

(b) $(\sin(\theta) + \cos(\theta))(\sin(\theta) + 1)$

(c) $\sin(2\theta) + 1$

(d) $\cos(2\theta) + 1$

(e) None of the above

13. Suppose the following statement is true:

    If either Jack or Jill is happy, then either Bob or Becky is unhappy.

    Suppose both Bob and Jack are unhappy. What can be concluded?

    (a) Jill and Becky are both happy.
    (b) Jill and Becky are both unhappy.
    (c) Jill is happy and Becky is unhappy.
    (d) Jill is unhappy and Becky is happy.
    (e) None of the above

14. Let $x > 0$ and consider the following data set S:

$$x, 2x, 3x, 6x$$

    Find the median for the data set S.

    (a) $2x$
    (b) $\dfrac{5x}{2}$
    (c) $3x$
    (d) $\dfrac{7x}{2}$
    (e) $4x$

15. Evaluate

$$\lim_{x\to\infty}\left(1+\frac{\ln(9)}{2x}\right)^{x}.$$

    (a) 1
    (b) $\ln(9)$
    (c) 3
    (d) $\dfrac{9}{2}$
    (e) The limit does not exist.

16. Find the number of solutions for

$$\sec(x) = -2x + 2$$

in the interval $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.

(a) 0
(b) 1
(c) 2
(d) 3
(e) There are no solutions.

17. Define $f(x) = \sin(x)$ on the domain $D = \left[\frac{3\pi}{2}, \frac{5\pi}{2}\right]$ and let $g(x) = f^{-1}(x)$. Calculate $g\left(\cos\left(0\right)\right)$.

(a) 0
(b) $\frac{\pi}{2}$
(c) $\frac{3\pi}{2}$
(d) 2π
(e) None of the above

18. Suppose f is a function on a domain D and assume S is an interval. Consider the following statement:

There exists x in D such that if y is in S, then $f(x) > y$.

If the above statement is false, then what can be concluded?

(a) If x is in D, then for some y in S, we have $f(x) \leq y$.
(b) If x is in D, then for all y in S, we have $f(x) \leq y$.
(c) For any x in D and any y in S, we have $f(x) \leq y$.
(d) For some x in D and for some y in S, we have $f(x) \leq y$.
(e) None of the above

19. Suppose a bag contains 10 blue marbles and 10 green marbles. If you randomly select 2 marbles from the bag, what is the probability that both marbles have the same color?

    (a) $\dfrac{9}{38}$

    (b) $\dfrac{1}{4}$

    (c) $\dfrac{9}{19}$

    (d) $\dfrac{1}{2}$

    (e) None of the above

20. Suppose we have the data list S given by

$$91, 77, 67, 85.$$

    How many times do we need to list the datum value $x = 90$ in S to guarantee that the mean of the data list is equal to 85?

    (a) 1

    (b) 2

    (c) 3

    (d) 4

    (e) None of the above

21. Evaluate $\sec(15^\circ)$.

    (a) $\sqrt{6} - \sqrt{2}$

    (b) $\sqrt{6} + \sqrt{2}$

    (c) $\dfrac{4}{\sqrt{6} - \sqrt{2}}$

    (d) $\dfrac{4}{\sqrt{2} - \sqrt{6}}$

    (e) None of the above

22. Suppose the sum of three times Bill's age and two times Martha's age is the same as the product of Jim's age squared minus one and Bill's age. If Martha is seven times older than Bill, find Jim's age rounded to the nearest whole number.

    (a) 3

    (b) 4

    (c) 5

    (d) 6

    (e) None of the above

23. Suppose a fair two-sided coin is flipped 100 times. Find the probability of getting heads at least twice.

    (a) $\dfrac{2^{100} - 100}{2^{100}}$

    (b) $\dfrac{2^{100} - 101}{2^{100}}$

    (c) $\dfrac{1}{2^{100}}$

    (d) $\dfrac{101}{2^{100}}$

    (e) None of the above

24. Suppose we have a right triangle with sidelengths a, b, and c with corresponding opposite angles $A = 30°$, $B = 60°$, and $C = 90°$. Find b in terms of a and c.

    (a) $b = \sqrt{a^2 + c^2}$

    (b) $b = \sqrt{a^2 - c^2}$

    (c) $b = \dfrac{c\sqrt{3}}{2}$

    (d) $b = \dfrac{c}{2}$

    (e) None of the above

25. Find the area of the region R bounded by $f(x) = \sqrt{25 - x^2}$ and $g(x) = |x| - 5$.

(a) $\dfrac{25(\pi + 1)}{2}$

(b) $\dfrac{25(\pi + 2)}{2}$

(c) $\dfrac{50(\pi + 1)}{2}$

(d) $\dfrac{50(\pi + 2)}{2}$

(e) None of the above

26. Suppose $f(x)$ is a real-valued function defined for every real number. If f satisfies the property

$$f(cx_1 + x_2) = c^2 f(x_1) + f(x_2), \quad \text{for any real numbers } x_1, x_2, c,$$

evaluate $f(9)$ when $f(1) = 2$.

(a) $f(9) = 18$

(b) $f(9) = 130$

(c) $f(9) = 162$

(d) $f(9) = 164$

(e) None of the above

27. Suppose the velocity (in feet per second) of a cat running along a street is given by $v(t) = 4 - t^2$. Find the total distance covered by the cat from $t = 0$ seconds to $t = 3$ seconds, rounded to the nearest whole number.

(a) 3 feet

(b) 5 feet

(c) 8 feet

(d) 9 feet

(e) None of the above

28. Suppose we have 10 different cards, labeled 1-10, and a game consists of turning over 1 of the 10 cards. If you turn over a prime number, you win \$7 and if not, you win nothing. What is the average winnings?

    (a) \$1.40
    (b) \$2.10
    (c) \$2.80
    (d) \$3.50
    (e) None of the above

29. Suppose 4 double-sided text cards are placed on a table in front of you labeled A, B, 1, and 2. If each card contains a letter on one side and a number on the other side, which cards need to be flipped over to guarantee the validity of the following statement:

    If B is on one side of a card, then 2 is on the other side of the same card.

    (a) A and 1
    (b) A and B
    (c) B and 1
    (d) B and 2
    (e) Only B needs to be flipped over.

30. Suppose a is a real number and define

$$f(x) = \begin{cases} x^a \sin\left(\frac{1}{x}\right) & \text{if } x > 0 \\ 0 & \text{if } x \leq 0. \end{cases}$$

    From the list below, what is the smallest value of a for which $f'(x)$ is continuous on $(-\infty, \infty)$?

    (a) $a = \frac{1}{2}$
    (b) $a = 1$
    (c) $a = \frac{3}{2}$
    (d) $a = 2$
    (e) $a = \frac{5}{2}$