Math Day 2023
at Murray State University
Lower Level Examination

- Do not open this exam until you are told to do so.
- Clearly fill in your NAME and STUDENT NUMBER on the bubble sheet. Your student number is located on the card your teacher gave you.
- You have 50 minutes to complete this exam.
- You may not use a calculator, phone, notes, book, or other aid. Any attempt to do so will result in disqualification.
- The exam will be scored as follows:

  +1 point for a correct answer
  $-\frac{1}{4}$ point for an incorrect answer
  0 points for a blank answer

- Clearly select one answer on the bubble sheet for each question. If more than one answer is selected, the answer will be marked as incorrect.

GOOD LUCK!

1. Find the distance between the lines $-2x + y = 1$ and $6x - 3y = 9$.

   (a) $\frac{4}{5}$

   (b) $\frac{2}{\sqrt{5}}$

   (c) $\frac{4}{\sqrt{5}}$

   (d) 4

   (e) None of the Above

2. The equation $x^2 - 2^x = 0$ has how many real solutions x?

   (a) 0

   (b) 1

   (c) 2

   (d) 3

   (e) 4

3. Suppose we have a pentagon such that the five interior angles have measures A, B, C, D, and E, respectively. If $A + B + C + D = 400°$, find the angle measure of E.

   (a) $50°$

   (b) $100°$

   (c) $120°$

   (d) $140°$

   (e) $180°$

4. If

$$x = \cfrac{1}{2 - \cfrac{1}{2 - \cfrac{1}{2 - \ldots}}}$$

then $x =$

(a) $\frac{1}{2}$

(b) $\frac{2}{3}$

(c) 1

(d) $\frac{3}{2}$

(e) None of the Above

5. Given that three of the roots of the polynomial $p(x) = a_5x^5+a_4x^4+a_3x^3+a_2x^2+a_1x+a_0$ are $2, i$, and $1+i$, find the sum of the remaining roots.

(a) $1+2i$

(b) $1-2i$

(c) $-1-2i$

(d) $-1+2i$

(e) 4

6. $\dfrac{5\sqrt{27}}{10-5\sqrt{3}} =$

(a) $\frac{3}{2}$

(b) 3

(c) $3(2\sqrt{3}+3)$

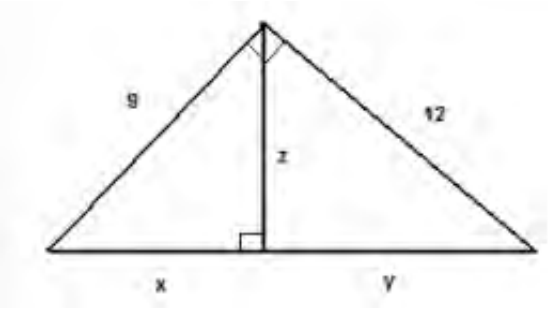
(d) $\dfrac{6\sqrt{3}+9}{7}$

(e) $\dfrac{6\sqrt{3}-9}{7}$

7. List the set of real solutions for the equation $\dfrac{x - 2\sqrt{x} - 3}{\sqrt{x} - 3} = 1$.

   (a) $\{0\}$
   (b) $\{9\}$
   (c) $\{0, 9\}$
   (d) The equation has no real solutions.
   (e) The solution set is all nonnegative real numbers except $x = 9$.

8. Consider the following figure which depicts three different right triangles:



   Find the length of side y.

   (a) y = 9.2
   (b) y = 9.4
   (c) y = 9.6
   (d) y = 9.8
   (e) y = 10

9. Suppose there is a bucket of marbles and Bill starts pulling them out in handfuls. He pulls out 1 marble on the first handful, and on each subsequent pull, he pulls out 2 more than he did on the previous pull. After 31 pulls, he notices that there are only 13 marbles left in the bucket. How many marbles were in the bucket before Bill started pulling them out?

   (a) 509
   (b) 961
   (c) 974
   (d) 1005
   (e) 1904

10. Jodi, Jewell, and Jazmine start unloading uniformly sized and weighted boxes from a full truck for which after 10 minutes, the truck is empty. In the past, Jodi has completed this exact task by herself in 45 minutes and Jewell has completed this exact task by herself in 40 minutes. Rounded to the nearest minute, how long would it take Jazmine to complete this exact task by herself?

   (a) 19 minutes
   (b) 23 minutes
   (c) 27 minutes
   (d) 30 minutes
   (e) 35 minutes

11. Suppose $x = \dfrac{\sqrt[3]{7}}{\sqrt[4]{13}}$. Then,

   (a) $x < 0$
   (b) $0 < x < 1$
   (c) $x = 1$
   (d) $1 < x < 2$
   (e) $2 \leq x$.

12. Consider the circle C given by $x^2 + y^2 = a^2$, where $a > 0$. Suppose we trace a path L starting at initial point $(a, 0)$, following counter-clockwise along the circle to $(0, a)$, then a line segment back to $(a, 0)$. If the area inside C is 100 cm^2, find the total length of L.

(a) $\dfrac{10 + 5\pi}{\pi}$ cm

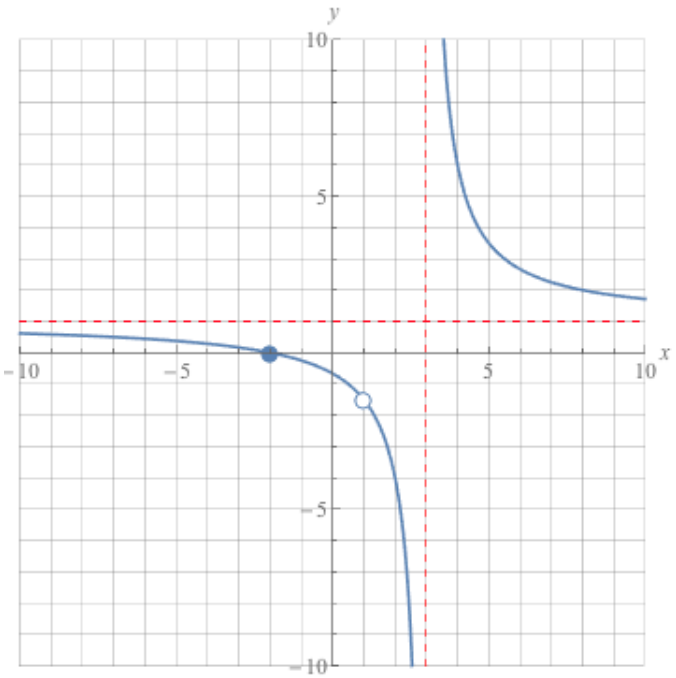
(b) $\dfrac{10 + 5\pi}{\sqrt{\pi}}$ cm

(c) $\dfrac{10\sqrt{2} + 5\pi}{\pi}$ cm

(d) $\dfrac{10\sqrt{2} + 5\pi}{\sqrt{\pi}}$ cm

(e) None of the Above

13. Suppose the graph of a rational function $f(x)$ is given below:



A possible formula for $f(x)$ is

(a) $f(x) = \dfrac{x + 2}{x - 3}$

(b) $f(x) = \dfrac{x + 2}{x^2 - 4x + 3}$

(c) $f(x) = \dfrac{x^2 - 4x + 3}{x^2 + x - 2}$

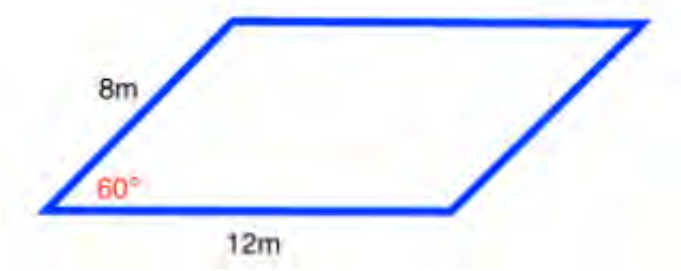
(d) $f(x) = \dfrac{x^2 + x - 2}{x^2 - 4x + 3}$

(e) $f(x) = \dfrac{x^2 + x - 2}{x - 3}$

14. The quadrant(s) where the function $f(x) = 16 - x^2$ intersects the line $x - y = -1$ are

(a) quadrants 1 and 2
(b) quadrants 2 and 4
(c) only quadrant 1
(d) only quadrant 3
(e) None of the Above

15. Find the area of the parallelogram given below.



(a) 32 m^2
(b) 48 m^2
(c) 96 m^2
(d) $32\sqrt{3}$ m^2
(e) $48\sqrt{3}$ m^2

16. If $x = (0.0000000243)^{\cdot 2}$, then

(a) $x = 0.0486$
(b) $x = 0.0000000486$
(c) $x = 0.03$
(d) $x = 0.0000000003$
(e) None of the Above

17. If $y = \dfrac{x^3 + 3x^2 + 2x + 1}{x^2 + 2x + 1}$, then

(a) $y = x^3 + 3$

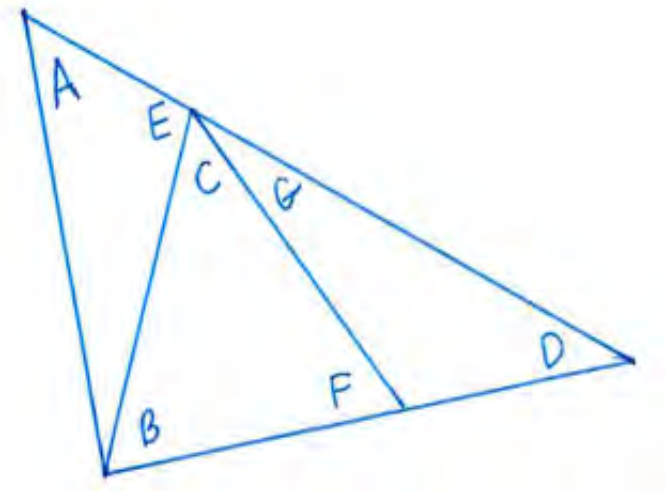
(b) $y = x + 1 - \dfrac{x}{(x+1)^2}$

(c) $y = x + 1 - \dfrac{x-1}{(x+1)^2}$

(d) $y = x - 1 - \dfrac{x}{(x+1)^2}$

(e) $y = x - 1 - \dfrac{x-1}{(x+1)^2}$

18. In the triangles given by the figure below, the angle measures of A, B, C, and D respectively are $70°, 60°, 50°$ and $30°$. The sum of the angle measures of E, F, and G is



(a) 180°

(b) 190°

(c) 200°

(d) 210°

(e) 220°

19. Find the sum $S = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ... + \frac{1}{512} + \frac{1}{1024}$.

(a) $S = \frac{511}{512}$

(b) $S = \frac{1023}{1024}$

(c) $S = \frac{2047}{2048}$

(d) $S = \frac{2049}{2048}$

(e) None of the Above

20. Given the circle $x^2 + y^2 = 9$, find the area A of any sector characterized by an arc length of π.

(a) $A = \frac{\pi}{2}$

(b) $A = \frac{3\pi}{2}$

(c) $A = \frac{9\pi}{2}$

(d) $A = \frac{27\pi}{2}$

(e) None of the Above

21. Simplify $\frac{(2,000,000,016)(8,000,000,004) - 64}{1,000,000,000}$.

(a) $16,000,000,000$

(b) $16,000,000,064$

(c) $16,000,000,072$

(d) $16,000,000,136$

(e) None of the Above

22. Suppose we have a square S with side s and we inscribe the largest circle C inside square S. If we then inscribe the largest such square S' inside circle C, find the area A of square S'.

(a) $A = \dfrac{s^2}{2}$

(b) $A = \dfrac{s^2}{4}$

(c) $A = \dfrac{s^2}{8}$

(d) $A = \dfrac{s^2}{16}$

(e) None of the Above

23. Find a decimal representation for the number $\dfrac{160}{99}$.

(a) 1.61

(b) $1.\overline{6}$

(c) $1.\overline{616}$

(d) $1.\overline{62}$

(e) None of the Above

24. Suppose we cut an arch segment from a circle such that the height of the arch is 4 and the width is 16. Find the radius r of the circle.

(a) $r = 8$

(b) $r = 10$

(c) $r = 12$

(d) $r = 16$

(e) None of the Above

25. Suppose $f(x) = \dfrac{x-2}{x+1}$. Simplify $(f \circ f)(x)$.

(a) $(f \circ f)(x) = \dfrac{x}{1-2x}$

(b) $(f \circ f)(x) = \dfrac{x+4}{1-2x}$

(c) $(f \circ f)(x) = \dfrac{x-4}{1-2x}$

(d) $(f \circ f)(x) = \dfrac{-x-4}{1-2x}$

(e) None of the Above

26. Suppose we have a triangle with sides a, b and c. If a line segment l is drawn perpendicular from side b connecting to the opposite vertex, then the length of l is 8. Given that the area of the triangle is 12, find a possible combination of lengths of sides a and c.

(a) $a = 7,\ c = 9$

(b) $a = 9,\ c = 12$

(c) $a = 9,\ c = 13$

(d) $a = 10,\ c = 10$

(e) None of the Above

27. If $x = 2 - 3 + 8 \div 2 \times 4$, then

(a) $x = -17$

(b) $x = -2$

(c) $x = 0$

(d) $x = 15$

(e) None of the Above

28. Let R be the region in the 1st quadrant bounded by the lines $x = 0$, $y = 0$, $x = 4$, and $y = 2x + 1$. What is the area A of the region R?

(a) $A = 18$

(b) $A = 20$

(c) $A = 22$

(d) $A = 24$

(e) None of the Above

29. Suppose in a bag, there are 2 red marbles, 2 blue marbles, 2 yellow marbles, and 2 pink marbles. If you reach in and grab 2 marbles at random, what is the probability of both marbles having the same color?

(a) $\frac{1}{8}$

(b) $\frac{1}{4}$

(c) $\frac{1}{2}$

(d) $\frac{4}{7}$

(e) None of the Above

30. Find the positive value of m so that the curves $y = mx + 3$ and $(x-3)^2 + (y-4)^2 = 4$ have exactly one intersection?

(a) $m = \frac{3 + 2\sqrt{6}}{10}$

(b) $m = \frac{6 + 2\sqrt{6}}{10}$

(c) $m = \frac{3 + 2\sqrt{6}}{5}$

(d) $m = \frac{6 + 2\sqrt{6}}{5}$

(e) None of the Above