Math Day 2023
at Murray State University
Upper Level Examination

- Do not open this exam until you are told to do so.
- Clearly fill in your NAME and STUDENT NUMBER on the bubble sheet. Your student number is located on the card your teacher gave you.
- You have 50 minutes to complete this exam.
- You may not use a calculator, phone, notes, book, or other aid. Any attempt to do so will result in disqualification.
- The exam will be scored as follows:

  +1 point for a correct answer
  $-\frac{1}{4}$ point for an incorrect answer
  0 points for a blank answer

- Clearly select one answer on the bubble sheet for each question. If more than one answer is selected, the answer will be marked as incorrect.

GOOD LUCK!

1. Suppose Smokey and Bandit start running down the same street along the same path. Bandit starts running at a speed of 15 mph. If Smokey starts at the same spot 10 seconds later at a speed of 20 mph, how far will Smokey have to run to catch Bandit?

   (a) $\frac{1}{6}$ miles

   (b) $\frac{1}{5}$ miles

   (c) $\frac{1}{4}$ miles

   (d) $\frac{1}{3}$ miles

   (e) None of the Above

2. Suppose a particle travels along the path of $y = x^2+x$, where both x and y are measured in inches. If the y-coordinate is changing at a rate of $4\frac{\text{inches}}{\text{second}}$ when the particle is at the point $(3, 12)$, find the rate of how the x-coordinate is changing at that moment.

   (a) $\frac{4}{7}\frac{\text{inches}}{\text{second}}$

   (b) $\frac{12}{7}\frac{\text{inches}}{\text{second}}$

   (c) $\frac{3}{2}\frac{\text{inches}}{\text{second}}$

   (d) $28\frac{\text{inches}}{\text{second}}$

   (e) $48\frac{\text{inches}}{\text{second}}$

3. Suppose the following statement is true:

   If Bella eats in the morning, then she will either take a nap in the afternoon or she will not eat in the afternoon.

   Which of the following is always a true statement?

   (a) If Bella both takes a nap in the afternoon and does not eat in the afternoon, then she ate in the morning.

   (b) If Bella does not take a nap in the afternoon, then she did not eat in the morning.

   (c) If Bella neither takes a nap nor eats in the afternoon, then she did not eat in the morning.

   (d) If Bella either does not take a nap in the afternoon or eats in the afternoon, then she did not eat in the morning.

   (e) None of the Above

4. Suppose a bag contains 5 red marbles, 7 blue marbles, and 8 black marbles. If you reach into the bag and randomly pull out 2 marbles, what is the probability that you will have a red marble and a blue marble?

   (a) $\frac{7}{80}$

   (b) $\frac{7}{76}$

   (c) $\frac{1}{10}$

   (d) $\frac{7}{40}$

   (e) $\frac{7}{38}$

5. Suppose that

$$3^x = \sqrt{3^x + \sqrt{3^x + \sqrt{3^x + \sqrt{3^x + \sqrt{3^x + ...}}}}}$$

Then $x =$

(a) $\log_3(2)$
(b) $\log_3(4)$
(c) $\log_2(3)$
(d) 1
(e) 2

6. Given that $0 < \theta < \dfrac{\pi}{2}$ and $\sin(\theta) = \dfrac{4}{5}$, evaluate $\sin\left(\theta + \dfrac{\pi}{3}\right)$.

(a) $\dfrac{2}{5}$
(b) $\dfrac{3\sqrt{3}}{10}$
(c) $\dfrac{4+3\sqrt{3}}{10}$
(d) $\dfrac{3+4\sqrt{3}}{10}$
(e) $\dfrac{8+5\sqrt{3}}{10}$

7. Suppose that currently at Murray State University, 20% of students have played intramural softball, 25% of students have played intramural football , and 10% have played both intramural softball and intramural football. If a random student at Murray State tells you that they have played intramural softball, what is the probability (as a percentage) that they have also played intramural football?

(a) 5%
(b) 10%
(c) 25%
(d) 45%
(e) 50%

8. If $6e^x - 5e^{-x} = -29$, then

   (a) $-10 < x \leq -5$

   (b) $-5 < x \leq 0$

   (c) $0 < x \leq 5$

   (d) $5 < x \leq 10$

   (e) None of the Above

9. Evaluate $\sec\left(\dfrac{146\pi}{12}\right)$.

   (a) $\dfrac{1}{2}$

   (b) $\sqrt{2}$

   (c) 2

   (d) $\dfrac{\sqrt{3}}{2}$

   (e) $\dfrac{2\sqrt{3}}{3}$

10. Suppose

$$x = 1 \cdot 3 \cdot 9 \cdot 27 \cdot 81 \cdot \ldots \cdot 3^{19} \cdot 3^{20}$$

    Then the ones digit of x is

    (a) 1

    (b) 3

    (c) 7

    (d) 9

    (e) None of the Above

11. Suppose $a, b > 0$. Evaluate $\sin^2\left(\tan^{-1}\left(\frac{a}{b}\right)\right)$.

(a) $\dfrac{\sqrt{a^2+b^2}}{a^2}$

(b) $\dfrac{a}{\sqrt{a^2+b^2}}$

(c) $\dfrac{a^2}{\sqrt{a^2+b^2}}$

(d) $\dfrac{a^2}{a^2+b^2}$

(e) $\dfrac{b^2}{a^2+b^2}$

12. Evaluate

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k^2}{n^3}$$

by interpreting as a Riemann sum.

(a) 0

(b) $\dfrac{1}{2}$

(c) $\dfrac{1}{3}$

(d) 1

(e) The limit does not exist.

13. Suppose both of the following statements are true:

I. If Joe becomes an engineer, he will never do any math.

II. If Joe does math, he will not be happy.

Suppose that Joe never does any math. What can always be concluded?

(a) Joe is happy.

(b) Joe will become an engineer.

(c) Joe will not become an engineer.

(d) Both (a) and (b) can be concluded.

(e) Nothing can be concluded.

14. Suppose the mean for an exam is 80% with standard deviation x. If Bill received an exam score of 88% which corresponds to a standardized z-score of 1.5, then what is the standard deviation x, rounded to the nearest whole percent?

    (a) $x \approx 5\%$
    (b) $x \approx 6\%$
    (c) $x \approx 8\%$
    (d) $x \approx 12\%$
    (e) $x \approx 15\%$

15. Suppose in a class, there are 4 parts to an overall grade: homework, worksheet, exam, and final exam. Each part of the grade is weighted according to the table below:

homework	20%
worksheet	10%
exam	40%
final exam	30%

    Suppose also you have a homework grade of 95%, a worksheet grade of 90%, and an exam grade of 80%. What is the minimum grade (on a scale from 0%-100%) you need on the final exam to achieve at least a 90% overall grade in the class?

    (a) 90%
    (b) 95%
    (c) 97%
    (d) 100%
    (e) It is impossible to receive at least a 90%.

16. Simplify the expression

$$\sin^2(x)\left[\frac{1+\cos^2(x)}{\cos^2(x)}+\cot^2(x)\right].$$

    (a) $\sin^2(x)$
    (b) $\csc^2(x)$
    (c) $\tan^2(x)$
    (d) $\cot^2(x)$
    (e) None of the Above

17. Evaluate $\lim_{x \to -\infty} x \sin(1/x)$.

(a) 0

(b) 1

(c) -1

(d) $-\infty$

(e) ∞

18. Find θ (in radians) so that $\csc(\theta) = -2$ and $\sec(\theta) < 0$.

(a) $\dfrac{13\pi}{6}$

(b) $\dfrac{17\pi}{6}$

(c) $\dfrac{19\pi}{6}$

(d) $\dfrac{23\pi}{6}$

(e) None of the Above

19. Consider the following:

Statement A: If x is a real number, then there exists a real number y such that $x < y$.

Which of the following statements is an equivalent statement to Statement A?

(a) If there exists a real number y such that that $y \leq x$, then x is not a real number.

(b) If there exists a real number y such that that $x < y$, then x is a real number.

(c) If for any real number y we have that $y \leq x$, then x is a real number.

(d) If for any real number y we have that $y \leq x$, then x is not a real number.

(e) None of the Above.

20. Suppose in a given triangle, we have angle measures (in degrees) A, B, and C with corresponding opposite sidelengths a, b and c (in inches), respectively. If $A = 60°$, $B = 45°$ and $c = 5$ inches, find the sum $a + b$.

(a) $a + b = \dfrac{10(\sqrt{3} + \sqrt{2})}{\sqrt{6} + \sqrt{2}}$ inches

(b) $a + b = \dfrac{10(\sqrt{6} + \sqrt{2})}{\sqrt{3} + \sqrt{2}}$ inches

(c) $a + b = \dfrac{10(\sqrt{3} + \sqrt{2})}{\sqrt{6} + \sqrt{3}}$ inches

(d) $a + b = \dfrac{10(\sqrt{6} + \sqrt{3})}{\sqrt{3} + \sqrt{2}}$ inches

(e) None of the Above

21. Consider the following list:

2,8,26,80,...

What is the next number in the sequence?

(a) 202

(b) 224

(c) 242

(d) 264

(e) None of the Above

22. Define $f(x) = x^x$. Find $f'(x)$ if it exists.

(a) x^x

(b) x^{x-1}

(c) $(x^2)^{x-1}$

(d) $x^x \ln(x)$

(e) None of the Above

23. Calculate the median for the following data set:

$$1, 13, 3, 38, 15$$

(a) 13

(b) 14

(c) 17.5

(d) 19.5

(e) None of the Above

24. Evaluate $\sin\left(\frac{\pi}{8}\right)$.

(a) $\sqrt{\frac{\sqrt{2}-1}{2}}$

(b) $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}}}$

(c) $\sqrt{\frac{1-\sqrt{2}}{2}}$

(d) $\sqrt{\frac{\sqrt{2}+1}{2}}$

(e) None of the Above

25. Find where the curve $y^2 = 2y - 1$ intersects $y = x^2 - 2x + 1$.

(a) $(2, -1)$

(b) $(-1, 2)$

(c) $(2, 0)$

(d) $(1, 2)$

(e) None of the Above

26. Suppose you play a game by rolling a standard 6-sided die. If it comes up a 6, you win \$100 and if it comes up a 5, you win \$50. Otherwise, you lose and win nothing. Find the average winnings for each play of this game.

(a) \$0

(b) \$25

(c) \$50

(d) \$75

(e) None of the Above

27. Let the recursion relation a_n satisfy

$$a_1 = 1,$$

$$a_{n+1} = 1 + 2a_n \quad \text{for} \quad n = 1, 2, 3, 4, ...$$

Evaluate $a_{100} - 2^{100}$.

(a) -1

(b) 1

(c) 2

(d) 2^{99}

(e) None of the Above

28. Let $F(t) = \int_0^t x^2 \, dx$. Evaluate $F(4)$.

(a) 8

(b) 16

(c) $\frac{64}{3}$

(d) 32

(e) None of the Above

29. Suppose exam scores follow a normal distribution with a mean of 80% and standard deviation of 7%. Given that a z-score of 2.33 coincides with the minimum exam score in the 99th percentile, find the minimum exam score in the 99th percentile (rounded to the nearest percentage point).

    (a) 96%

    (b) 97%

    (c) 98%

    (d) 99%

    (e) None of the Above

30. Suppose the sale price of a car is $\$20,000$ which is 10% off the original price. What was the original price?

    (a) $\$18,000$

    (b) $\$22,000$

    (c) $\$22,500$

    (d) $\$24,000$

    (e) None of the Above