Math Day 2024
at Murray State University
Lower Level Examination

- Do not open this exam until you are told to do so.
- Clearly fill in your NAME and STUDENT NUMBER on the bubble sheet. Your student number is located on the card your teacher gave you.
- You have 50 minutes to complete this exam.
- You may not use a calculator, phone, notes, book, or other aid. Any attempt to do so will result in disqualification.
- The exam will be scored as follows:

  +1 point for a correct answer
  $-\frac{1}{4}$ point for an incorrect answer
  0 points for a blank answer

- Clearly select one answer on the bubble sheet for each question. If more than one answer is selected, the answer will be marked as incorrect.

GOOD LUCK!

1. If an equilateral triangle has a height of 10, what is the area of the triangle?

   (a) $\dfrac{50}{\sqrt{3}}$

   (b) 50

   (c) $\dfrac{100}{\sqrt{3}}$

   (d) $\dfrac{200}{\sqrt{3}}$

   (e) None of the above

2. Suppose an empty swimming pool needs to be filled with water. Pump A can fill the pool in 3 hrs and pump B can fill the pool in 4 hours. If pump A is turned on at 12:00 noon and pump B is turned on at 1:30 pm, then rounding up to the nearest minute, what is the earliest time that the pool will be filled?

   (a) 2:20 pm

   (b) 2:21 pm

   (c) 2:22 pm

   (d) 2:23 pm

   (e) None of the above

3. Let L_1 be the line given by the equation $6x + 2y = 40$. If L_2 is the perpendicular line to L_1 at the x-intercept of L_1, find the y-intercept of L_2.

   (a) $\left(0, \dfrac{20}{9}\right)$.

   (b) $\left(0, \dfrac{20}{3}\right)$.

   (c) $\left(0, -\dfrac{20}{3}\right)$.

   (d) $(0, -20)$.

   (e) None of the above

4. Suppose you wish to create a square array of coins that is as large as possible. If you have 1950 coins, how many coins are not used in making this square array?

   (a) 10
   (b) 14
   (c) 18
   (d) 25
   (e) None of the above

5. Let R be the region of the coordinate plane consisting of those points (x, y) for which $x - 5 \leq y \leq 4 - x$ and $0 \leq x \leq 3$. What is the area of R?

   (a) 12
   (b) 15
   (c) 20
   (d) 60
   (e) None of the above

6. If $\{a_n\}$ is a positive sequence satisfying $a_{n+1}^2 = a_n^2 + 6a_n - 4a_{n+1} + 5$ for $n \geq 1$ and $a_1 = 1$, then determine a_{1000}.

   (a) 1000
   (b) 2000
   (c) 3000
   (d) 4000
   (e) None of the above

7. If x_0 is the smallest positive solution to $3x^3 - 10x^2 - 81x + 28 = 0$, then

   (a) $0 < x_0 < 1$
   (b) $1 \leq x_0 < 2$
   (c) $2 \leq x_0 < 3$
   (d) $3 \leq x_0 < 4$
   (e) $4 \leq x_0$

8. The fraction $\frac{5}{7}$ has which of the following as a decimal representation?

   (a) $0.\overline{714}$

   (b) $0.\overline{7142}$

   (c) $0.\overline{71428}$

   (d) $0.\overline{714285}$

   (e) $0.\overline{7142857}$

9. Suppose you wish to know the height of a building. You look up with angle of elevation 60° and can see the top of the building. If you are standing 200 feet from the building, then from the choices below, which best describes the height of the building?

   (a) 100 feet

   (b) $100\sqrt{2}$ feet

   (c) $100\sqrt{3}$ feet

   (d) $200\sqrt{3}$ feet

   (e) $\frac{200}{\sqrt{3}}$ feet

10. If both of the ordered pairs $(1, 4)$ and $(2, 2)$ lie on a circle with center $\left(\frac{1}{6}, k\right)$, then what is the radius r of the circle?

    (a) $\frac{\sqrt{5}}{2}$

    (b) $\frac{5}{6}$

    (c) $\frac{5\sqrt{5}}{6}$

    (d) $\frac{121}{36}$

    (e) None of the above

11. Suppose at a store, a pair of shoes is on sale for \$102. If the discount was %15 and x represents the original price of the shoes, then which of the following is satisfied?

    (a) $\$115 < x \leq \$117$

    (b) $\$117 < x \leq \$119$

    (c) $\$119 < x \leq \$121$

    (d) $\$121 < x \leq \$123$

    (e) None of the above

12. The graphs of $x = \dfrac{1}{y}$ and $y = x + 10$ have how many intersection points?

    (a) 0

    (b) 1

    (c) 2

    (d) 3

    (e) 4

13. Four pipes are stacked as shown below, with each pipe having a diameter of 4 inches. What is the diameter of the largest possible pipe that would fit within the space enclosed by the four 4-inch-diameter pipes?

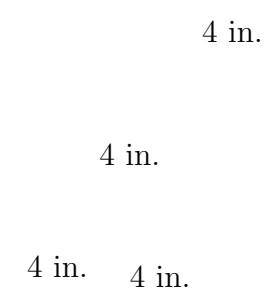
    (a) $2\sqrt{2}$

    (b) $2(\sqrt{2}+1)$

    (c) $4(\sqrt{2}-1)$

    (d) 4

    (e) None of the above



14. The smallest leg of a right triangle has endpoints with coordinates $(1, 1)$ and $(2, 3)$ in the xy-plane. What is the ordered pair in the 4th quadrant with integer coordinates that yields the smallest right triangle?

    (a) $(8, -1)$
    (b) $(9, -1)$
    (c) $(11, -2)$
    (d) $(12, -2)$
    (e) None of the above

15. What is the area of the triangle with vertices $(1, 1), (3, 2)$, and $(2, 4)$?

    (a) $\dfrac{\sqrt{5}}{2}$
    (b) $\dfrac{5}{2}$
    (c) 2
    (d) 3
    (e) None of the above

16. The solution set of the equation $\dfrac{2x^3 - 8x^2 + 8x}{(x-2)^2} = 2x$ is

    (a) $\{0\}$
    (b) All real numbers
    (c) All real numbers except 2
    (d) All real numbers except -2 and 2
    (e) None of the above

17. Find an equivalent expression for the perimeter of the parallelogram P where the four vertices of P are characterized by the ordered pairs $(3, 4)$, $(7, 5)$, $(4, 7)$, and $(8, 8)$.

(a) 14

(b) 18

(c) $\dfrac{7}{\sqrt{17} - \sqrt{10}}$

(d) $\dfrac{14}{\sqrt{17} - \sqrt{10}}$

(e) $\dfrac{18}{\sqrt{17} - \sqrt{10}}$

18. Suppose we have 4 groups of people, group A, group B, group C, and group D. Also, suppose all of the following statements are true:

- All of group B is in group A.
- Some of group C is in group B.
- None of group D is in group A.

What must be true?

(a) There cannot be anyone in both groups B and D.

(b) Some of group C is in group D.

(c) Anyone in both groups A and C are in group B.

(d) All of group C is either in group A or group D.

(e) None of the above

19. If $f(x) = \dfrac{x+3}{2x-1}$, then $(f \circ f)(x)$ is

(a) $\dfrac{x+6}{2x-5}$

(b) $\dfrac{7x}{5}$

(c) x

(d) $\left(\dfrac{x+3}{2x-1}\right)^2$

(e) None of the above

20. In the parallelogram $\lozenge ABCD$, sides $\overline{AB}$ and $\overline{CD}$ have length 15, sides $\overline{AD}$ and $\overline{BC}$ have length 7, angle $\angle DAB$ is obtuse, and angle $\angle DAC$ is a right angle. What is the area of the parallelogram?

(a) 105
(b) $28\sqrt{11}$
(c) $60\sqrt{11}$
(d) $7\sqrt{274}$
(e) $15\sqrt{274}$

21. Suppose that the points $P = (0, 2)$ and $Q = (1, 0)$ are endpoints for a side of a square $\square PQRS$ whose other vertices lie in Quadrant I. If S is the vertex of the square with maximum y-value, then S is

(a) $\left(\frac{5}{4}, \frac{21}{8}\right)$
(b) $\left(\frac{3}{2}, \frac{11}{4}\right)$
(c) $\left(\frac{7}{4}, \frac{23}{8}\right)$
(d) $(2, 3)$
(e) None of the above

22. A circle in the coordinate plane has its center on the positive part of the x-axis and passes through the points $(0, 5)$ and $(-2, 0)$. The radius of the circle is

(a) $\frac{21}{4}$
(b) $\frac{25}{4}$
(c) $\frac{29}{4}$
(d) $\frac{33}{4}$
(e) None of the above

23. How many x-intercepts does the function $R(x) = \dfrac{x^3 + 7x^2 + 2x - 40}{x^2 - 5x + 6}$ have?

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

24. In a right triangle, assume the length of the adjacent leg to the angle θ is 4 and the length of the opposite leg is 5. Evaluate $\sin(\theta)\tan(\theta)$.

(a) $\dfrac{20}{\sqrt{41}}$

(b) $\dfrac{20}{41}$

(c) $\dfrac{25}{4\sqrt{41}}$

(d) $\dfrac{4}{\sqrt{41}}$

(e) None of the above

25. Given that $a_{n+1} = 1 + 2a_n$ for $n \geq 1$ and that $a_{100} = 2^{100} - 1$, find a_1.

(a) 1

(b) 3

(c) 7

(d) 15

(e) 31

26. Simplify the following:

$$\left(\sqrt{5}\right)^{\log_2(4)} + \left(\left(\sqrt[3]{7}\right)^{\sqrt[3]{3}}\right)^{\sqrt[3]{9}}$$

(a) $5^8 + \sqrt[3]{189}$

(b) $5 + \sqrt[3]{189}$

(c) $5^8 + 7$

(d) 12

(e) None of the above

27. If $x = \cfrac{3}{9 + \cfrac{3}{9 + \cfrac{3}{9 + \cfrac{3}{\ddots}}}}$, then $x =$

(a) $\dfrac{1}{3}$

(b) $\dfrac{1}{4}$

(c) $\dfrac{-9 + \sqrt{83}}{2}$

(d) $\dfrac{-9 + \sqrt{85}}{2}$

(e) $\dfrac{-9 + \sqrt{93}}{2}$

28. If $x = \sqrt[5]{7}$ and $y = \sqrt[4]{5}$, then

(a) $1 < x < y < 2$

(b) $2 < x < y < 3$

(c) $1 < y < x < 2$

(d) $2 < y < x < 3$

(e) None of the above

29. What is the ones digit of $2^{10^{10}}$?

(a) 0

(b) 2

(c) 4

(d) 6

(e) 8

30. Suppose 4 cats and 3 dogs are randomly placed in a line. What is the probability that no two cats are sitting next to one another?

(a) $\frac{3!}{4!}$

(b) $\frac{4!}{7!}$

(c) $\frac{3!}{7!}$

(d) $\frac{3!4!}{7!}$

(e) None of the above