## Math Day 2025
at Murray State University
Lower Level Examination

- Do not open this exam until you are told to do so.

- Clearly fill in your NAME and STUDENT NUMBER on the bubble sheet. Your student number is located on the card your teacher gave you.

- You have 60 minutes to complete this exam.

- You may not use a calculator, phone, notes, book, or other aid. Any attempt to do so will result in disqualification.

- The exam will be scored as follows:

  $+1$ point for a correct answer

  $-\frac{1}{4}$ point for an incorrect answer

  0 points for a blank answer

- Clearly select one answer on the bubble sheet for each question. If more than one answer is selected, the answer will be marked as incorrect.

GOOD LUCK!

1. The circles shown are tangent at point B. Point A is the center of the larger circle, and line segment $\overline{AB}$ is a diameter of the smaller circle. If the area of the larger circle is equal to 3, what is the area of the smaller circle?
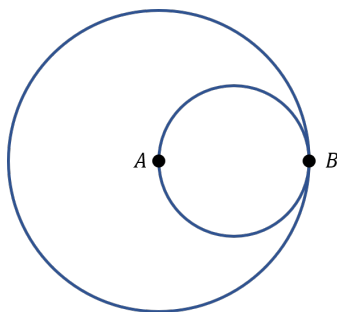
  (a) 1/2
  (b) 3/4
  (c) $3/\pi$
  (d) 1
  (e) None of the above

2. Suppose the original price x of a shirt was raised by 10%, but you purchase the shirt with a 5% off coupon. How much did you pay for the shirt?

  (a) $0.95x$
  (b) $1.045x$
  (c) $1.05x$
  (d) $1.055x$
  (e) None of the above

3. A car got 30 miles per gallon using gasoline that cost $2.75 per gallon. What was the approximate cost, in dollars, of the gasoline used in driving the car 500 miles?

  (a) $40
  (b) $42
  (c) $44
  (d) $46
  (e) None of the above

4. A certain jar contains 50 marbles: 15 white, 14 green, 10 yellow, 6 red, and 5 purple. If one marble is chosen from the jar at random, what is the probability that the marble will be neither red nor purple?

   (a) 0.22

   (b) 0.39

   (c) 0.44

   (d) 0.78

   (e) None of the above

5. The figure below illustrates two concentric circles with center A where $|\overline{AB}| = 2|\overline{BC}|$.
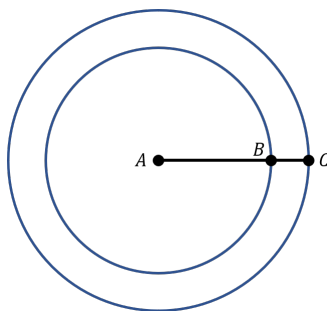
   If the area of the circle with center A defined by the radius $\overline{AB}$ is equal to 10, find the area of the circle with center A defined by radius $\overline{AC}$.

   (a) 12

   (b) 15

   (c) 18

   (d) 20

   (e) None of the above

6. What is the number of intersection points of $f(x) = |2x - 4|$ and $g(x) = x$ on the common domain $(-\infty, \infty)$?

   (a) 0

   (b) 1

   (c) 2

   (d) 3

   (e) None of the above

7. Which of the following could NOT be the ones digit of 562^n, where n is a positive integer?

   (a) 0

   (b) 2

   (c) 4

   (d) 6

   (e) None of the above

8. In which quadrant or quadrants does the solution set of the system

$$y - 4x < 3$$

$$y < -x^2 - 6x - 7$$

   lie?

   (a) quadrant I

   (b) quadrant II

   (c) quadrant III

   (d) quadrants II and III

   (e) None of the above

9. Working alone at its constant rate, a new machine produces 5 wheels in 15 minutes. Working alone at its constant rate, an older machine produces 5 wheels in 24 minutes. Working together simultaneously, how many minutes does it take both machines to produce 100 wheels, rounded to the nearest minute?

    (a) 185 minutes

    (b) 195 minutes

    (c) 390 minutes

    (d) 780 minutes

    (e) None of the above

10. Suppose general ticket prices to a basketball game are \$15 and student ticket prices are \$5. If there were exactly 5000 combined tickets sold for which \$60,000 was generated, then the number of student tickets x satisfies which of the following?

    (a) $1 \le x < 1000$

    (b) $1001 \le x < 2000$

    (c) $2001 \le x < 3000$

    (d) $3001 \le x < 4000$

    (e) None of the above

11. Suppose $x > 0$. In the figure below, the line segments $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$ satisfy the proportion
$$\frac{|\overline{AC}|}{|\overline{AB}|} = \frac{|\overline{AB}|}{|\overline{BC}|}.$$

Determine the length $|\overline{AB}|$ given that $|\overline{BC}| = x$.

    (a) $2x$

    (b) $\sqrt{3}x$

    (c) $\dfrac{1 - \sqrt{5}}{2}x$

    (d) $\dfrac{1 + \sqrt{5}}{2}x$

    (e) None of the above

12. A highly-secured pass-code to enter a top-secret facility consists of the digits 0-9 only, no digit may be repeated more than once, and the code is 6 digits long. How many pass-codes are possible?

   (a) $6!$

   (b) $\dfrac{10!}{6!}$

   (c) $\dfrac{9!}{6!}$

   (d) $\dfrac{10!}{4!}$

   (e) None of the above

13. Suppose we have a deck of 32 cards which each contain one of the following letters A,B,C,D,E,F,G,H. Each letter is on exactly four cards in the deck. If two cards are drawn from the deck without replacement, what is the probability of getting two cards of the same letter?

   (a) $\dfrac{3}{248}$

   (b) $\dfrac{1}{64}$

   (c) $\dfrac{3}{31}$

   (d) $\dfrac{1}{8}$

   (e) None of the above

14. How many real solutions does the following equation have?

$$x(x-3)(x+7) - 26 = x^3 + (2x-1)^2 - 17(x+1) - 10$$

   (a) 0

   (b) 1

   (c) 2

   (d) An infinite number

   (e) None of the above

15. Suppose that in a lab, solution A is 50% hydrochloric acid, while solution B is 75% hydrochloric acid. Using only these two solutions, how many liters of solution A should be used to make 100 liters of a new solution which is 60% hydrochloric acid?

    (a) 56
    (b) 58
    (c) 60
    (d) 62
    (e) None of the above

16. Suppose you wish to know the height of a tree. You are standing 100 feet from the tree and can see the top of the tree with angle of elevation 30°. If you are 6 feet tall, what is the approximate height of the tree?

    (a) 39 feet
    (b) $100\sqrt{3} + 6$ feet
    (c) $\dfrac{100 + 6\sqrt{2}}{\sqrt{2}}$ feet
    (d) $\dfrac{100 + 6\sqrt{3}}{\sqrt{3}}$ feet
    (e) None of the above

17. Let $a > 0$ and $2^x = a$. Simplify $y = \dfrac{(4^x + 16^{2x})^2}{8^{3x}}$.

    (a) $y = a^7 + 2a + a^{-5}$
    (b) $y = a^6 + 2 + a^{-6}$
    (c) $y = a^5 + 2a^{-1} + a^{-7}$
    (d) $y = a^4 + 2a^{-2} + a^{-8}$
    (e) None of the above

18. What is the sum S of the interior angles for a pentagon?

(a) $S = 360°$

(b) $S = 540°$

(c) $S = 720°$

(d) $S = 900°$

(e) None of the above

19. If s represents the positive solution to $3x^4 = x^2 + 10$, then s satisfies which of the following?

(a) $0 < s < 1$

(b) $1 < s < 2$

(c) $2 < s < 3$

(d) $3 < s < 4$

(e) None of the above

20. Find the $10th$ term in the sequence characterized by the list $\{6, 18, 36, 60, 90, ...\}$.

(a) 180

(b) 216

(c) 252

(d) 324

(e) None of the above

21. In how many distinct ways can a five number pass-code be formed consisting of exactly three 0's and two 1's?

(a) 7

(b) 8

(c) 9

(d) 10

(e) None of the above

22. Suppose the following statement is true:

"If Jimmy is hungry, then he either eats fish or chicken".

Which of the following statements must be true?

(a) "If Jimmy eats fish or chicken, then he is hungry".

(b) "If Jimmy isn't hungry, then he neither eats fish nor chicken".

(c) "If Jimmy doesn't eat fish or chicken, then he isn't hungry".

(d) "If Jimmy neither eats fish nor chicken, then he isn't hungry".

(e) None of the above.

23. A recursive relation a_n is defined as $a_{n+1} = n+1+a_n$ when $n \geq 1$. If $a_1 = 1$, determine a_{500}.

(a) 124750

(b) 125250

(c) 249500

(d) 250500

(e) None of the above

24. If $0 < st < 1$, then which of the following must be true?

(a) $0 < s < 1$ or $0 < t < 1$

(b) $0 < s < 1$ or $1 < t$

(c) $-1 < s < 0$ or $-1 < t < 0$

(d) $s > -1$ and $t > -1$

(e) None of the above

25. At a store, one soda costs 50 cents and a 12-pack of sodas costs $5.25. What is the largest amount of sodas you can purchase with $110?

(a) 220

(b) 240

(c) 250

(d) 252

(e) None of the above

26. Two machines S and T produce 1000 and 2000 auto parts every day, respectively. On average, Machine S produces 10 defective parts and machine T produces 40 defective parts each day. What is the probability that a randomly selected part will be defective?

   (a) 1/3
   (b) 1/5
   (c) 3/25
   (d) 1/60
   (e) None of the above

27. If a and b are the roots of the equation $2x^2 - 9x + 8 = 0$, then the product ab is equal to

   (a) -2
   (b) 4
   (c) 49/2
   (d) 32
   (e) None of the above

28. If $\mathscr{L}$ is the equation of the line perpendicular to $x + y = 10$ that passes through the point $(-3, 15)$, find the x-intercept of $\mathscr{L}$.

   (a) $x = -16$
   (b) $x = -12$
   (c) $x = 10$
   (d) $x = 18$
   (e) None of the above

29. In the figure, O is the center of the circle and P, Q, and R are on the circle where O is contained inside of $\triangle PQR$. Given that $\angle QPO + \angle OPR = 52°$, then $\angle QOR$ is
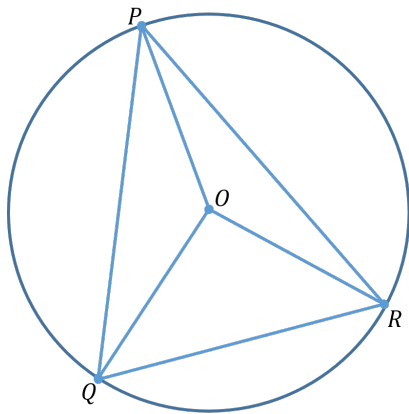
   (a) 98°

   (b) 100°

   (c) 102°

   (d) 104°

   (e) None of the above

30. What is the greatest distance between two vertices of a rectangular solid with a height of 10, a length of 4, and a volume of 120?

   (a) $2\sqrt{29}$

   (b) 5

   (c) $5\sqrt{5}$

   (d) $\sqrt{149}$

   (e) None of the above