Math Day 2025
at Murray State University
Upper Level Examination

- Do not open this exam until you are told to do so.
- Clearly fill in your NAME and STUDENT NUMBER on the bubble sheet. Your student number is located on the card your teacher gave you.
- You have 60 minutes to complete this exam.
- You may not use a calculator, phone, notes, book, or other aid. Any attempt to do so will result in disqualification.
- The exam will be scored as follows:

  +1 point for a correct answer
  $-\frac{1}{4}$ point for an incorrect answer
  0 points for a blank answer

- Clearly select one answer on the bubble sheet for each question. If more than one answer is selected, the answer will be marked as incorrect.

GOOD LUCK!

1. Suppose points P and Q have a distance from one another of 200 miles on a given highway. Assume that at 1:00 pm, Pat enters the highway at point P traveling towards point Q at 65 mph and Tom enters the highway at a point Q traveling towards point P at 55 mph. If both speeds remain constant, at what time will Pat meet Tom?

   (a) 2:30 pm
   (b) 2:40 pm
   (c) 2:45 pm
   (d) 3:00 pm
   (e) None of the above

2. Evaluate the integral

$$I = \int_0^3 \left(\sqrt{9 - x^2} + x\right)\, dx.$$

   (a) $I = \dfrac{9\pi + 9}{2}$
   (b) $I = \dfrac{9\pi + 18}{2}$
   (c) $I = \dfrac{9\pi + 18}{4}$
   (d) $I = \dfrac{9\pi + 36}{4}$
   (e) None of the above

3. Suppose the following statement is false:

   They like math, but they neither like history nor science.

   Which of the following is always a true statement?

   (a) If they don't like math, then they either like history or science.
   (b) If they like math, then they either like history or science.
   (c) If they don't like math, then they neither like history nor science.
   (d) If they like math, then they neither like history nor science.
   (e) They don't like math, but they either like history or science.

4. Suppose that a 2-sided coin has possible outcomes 1 and 2, each with equal probability. If the coin is flipped 5 times, find the probability that you get a 1 on at least 2 flips.

   (a) $\frac{3}{16}$

   (b) $\frac{1}{4}$

   (c) $\frac{2}{5}$

   (d) $\frac{13}{16}$

   (e) None of the above

5. Suppose that

$$x = \frac{10}{3 + \frac{10}{3 + \frac{10}{3 + \ldots}}}.$$

   Then

   (a) $x = 2$

   (b) $x = 3$

   (c) $x = 4$

   (d) $x = 5$

   (e) None of the above

6. Let $x \geq 0$. Calculate $\cos\left(\tan^{-1}\left(x + 2\right)\right)$.

   (a) $\frac{\sqrt{x^2 + 4x + 5}}{x + 2}$.

   (b) $\frac{x + 2}{\sqrt{x^2 + 4x + 5}}$.

   (c) $x + 2$

   (d) $\frac{1}{x + 2}$.

   (e) None of the above

7. Suppose the heights of American males are normally distributed with a mean of 68 inches and standard deviation of 4 inches. If an American male is chosen randomly, what is the approximate probability that he will have a height of 72 inches or less?

   (a) .74

   (b) .79

   (c) .84

   (d) .89

   (e) None of the above

8. Simplify the following expression when $x \neq 3$:

$$\frac{x^4 + 5x^3 - 29x^2 - 69x + 252}{x - 3}$$

   (a) $x^3 - 8x^2 + 5x + 84$

   (b) $x^3 + 8x^2 - 5x - 84$

   (c) $x^3 - 8x^2 - 5x + 84$

   (d) $x^3 - 8x^2 - 5x - 84$

   (e) None of the above

9. An Earth satellite in circular orbit 1200 km high makes one complete revolution every 152 minutes. What is the linear speed of the satellite? Use 6400 km for the radius of the Earth.

   (a) $50\pi \dfrac{km}{minute}$

   (b) $100\pi \dfrac{km}{minute}$

   (c) $150\pi \dfrac{km}{minute}$

   (d) $200\pi \dfrac{km}{minute}$

   (e) None of the above

10. Calculate the following sum:

$$S = 5 + 10 + 15 + 20 + 25 + ... + 995 + 1000$$

(a) $S = 99500$

(b) $S = 100000$

(c) $S = 100500$

(d) $S = 101000$

(e) None of the above

11. Suppose the position of a particle follows the position function $s(t) = -(t-10)^2 + 400$. Evaluate the speed at the instant when the particle hits the ground.

(a) speed $= -40$

(b) speed $= 0$

(c) speed $= 10$

(d) speed $= 30$

(e) None of the above

12. If

$$x = \frac{\sin^2(\theta) + \cos^2\left(\frac{\theta}{2}\right)}{\sin(\theta)},$$

then

(a) $x = \frac{1}{2}(\csc(\theta) + \cot(\theta)) + \sin(\theta)$

(b) $x = \frac{1}{2}(\sin(\theta) + \cot(\theta)) + \csc(\theta)$

(c) $x = \sin(\theta) + \frac{1}{2}\cot(\theta)\cos(\theta)$

(d) $x = \csc(\theta)$

(e) None of the above

13. Suppose the following statement is true:

    If both Jack and Jill are happy, then either it is sunny or it is not raining.

    Suppose it is either not sunny or it is raining. What can be concluded?

    (a) Jack and Jill are both unhappy.
    (b) Either Jack or Jill is unhappy.
    (c) Jack and Jill are both happy.
    (d) Either Jack or Jill is happy.
    (e) None of the above

14. Let $x > 0$ and consider the following data set S:

    $$x, 2x, 3x, 6x$$

    Find the sample standard deviation for the data set S.

    (a) $\frac{3}{2}x$
    (b) $3x$
    (c) $\sqrt{\frac{7}{2}}x$
    (d) $\sqrt{\frac{14}{3}}x$
    (e) None of the above

15. Evaluate

    $$\lim_{x\to\infty} \sqrt{x^2+4x} - \sqrt{x^2-2x}.$$

    (a) 0
    (b) $\sqrt{6}$
    (c) 3
    (d) 6
    (e) The limit does not exist.

16. Find the number of solutions for

$$\sin(4x) = -\frac{1}{2}$$

in the interval $\left(\frac{\pi}{4}, 4\pi\right)$.

(a) 6

(b) 8

(c) 14

(d) 16

(e) None of the above

17. Define $f(x) = \sec(x)$ on the domain $D = \left(\frac{5\pi}{2}, 3\pi\right]$ and let $g(x) = f^{-1}(x)$. Calculate $g\left(\sec\left(\pi\right)\right)$.

(a) π

(b) 2π

(c) 3π

(d) $g\left(\sec\left(\pi\right)\right)$ is undefined

(e) None of the above

18. Let $S = \{1, 2, 3, 4\}$ and $R = \{5, 6, 7, 8\}$. Suppose there are four cards on a table which each have an element from S on one side of the card and an element from R on its other side. Also suppose we have

Rule: If 3 is on one side, then 7 must be on its other side.

If the four cards on the table show $2, 3, 7$, and 8, which cards must be flipped over to be sure that the Rule is obeyed?

(a) The cards 3 and 7 must be flipped over.

(b) The cards 3 and 8 must be flipped over.

(c) The cards 3,7 and 8 must be flipped over.

(d) All four cards must be flipped over.

(e) None of the above

19. Suppose a bag contains 4 blue marbles, 4 red marbles, and 4 green marbles. If you randomly select 3 marbles from the bag, what is the probability that you get a marble of each color?

    (a) $\dfrac{8}{165}$

    (b) $\dfrac{16}{165}$

    (c) $\dfrac{16}{55}$

    (d) $\dfrac{32}{55}$

    (e) None of the above

20. Suppose we have the data list S given by

$$1, 2, 3, 4, 5, 6, 7, 8.$$

    How many times do we need to list the datum value $x = 50$ in S to guarantee that the mean of the data list is equal to 46?

    (a) 56

    (b) 65

    (c) 74

    (d) 83

    (e) None of the above

21. Evaluate $\sin(15^\circ)$.

    (a) $\dfrac{\sqrt{6} - 2\sqrt{2}}{2}$

    (b) $\dfrac{\sqrt{6} + 2\sqrt{2}}{2}$

    (c) $\dfrac{2\sqrt{2} - \sqrt{6}}{2}$

    (d) $\dfrac{\sqrt{6} + 2\sqrt{2}}{4}$

    (e) None of the above

22. Suppose 10,000 people attend a basketball game where ticket sales reach $\$189,000$. Also suppose everybody at the game paid for a ticket and there were only two types of tickets sold, student tickets and general tickets. If student tickets were \$5 and general tickets were \$25, how many student tickets were bought?

    (a) 2780

    (b) 3050

    (c) 3275

    (d) 3375

    (e) None of the above

23. Suppose a fair six-sided die with sides 1,2,3,4,5, and 6 is rolled 10 times. Find the probability of getting either a 5 or 6 on at least one roll.

    (a) $\dfrac{3^{10} - 2^{10}}{3^{10}}$

    (b) $\dfrac{2^{10}}{3^{10}}$

    (c) $\dfrac{1}{3}$

    (d) $\dfrac{1}{3^{10}}$

    (e) None of the above

24. Suppose $a > 0$. Find the area inside an equilateral triangle with sidelength a.

    (a) area $= \dfrac{1}{2}a^2$

    (b) area $= \dfrac{1}{4}a^2$

    (c) area $= \dfrac{\sqrt{3}}{2}a^2$

    (d) area $= \dfrac{\sqrt{3}}{4}a^2$

    (e) None of the above

25. Suppose $f(x), g(x)$, and $h(x)$ are all differentiable functions everywhere. Calculate

$$j(x) = \frac{d}{dx}(f \circ g \circ h)(x).$$

(a) $j(x) = (f' \circ g \circ h)h'(x)$

(b) $j(x) = (f' \circ g \circ h)(g' \circ h)h'(x)$

(c) $j(x) = (f' \circ g \circ h)(f \circ g' \circ h)(f \circ g \circ h')(x)$

(d) $j(x) = (f' \circ g \circ h + f \circ g' \circ h + f \circ g \circ h')(x)$

(e) None of the above

26. Suppose $f(x)$ is a real-valued function defined for every real number and let $c > 0$. If f satisfies the property

$$f(cx_1 + x_2) = cf(x_1) + f(x_2) - c \quad \text{for any real numbers } x_1, x_2,$$

evaluate $f(0)$.

(a) $f(0) = 0$

(b) $f(0) = c$

(c) $f(0) = -c$

(d) There is not enough information to compute $f(0)$.

(e) None of the above

27. Suppose 2 cats are at the exact same point P and start running at the same time $t = 0$. One cat starts running east with velocity (in feet per second) $v_1(t) = t^{1/2}$ and the other cat starts running north with velocity (in feet per second) $v_2(t) = t^{3/2}$. If $x(t)$ represents the distance (in feet) between the 2 cats at time t (in seconds), evaluate $x(5)x'(5)$.

(a) $x(5)x'(5) = 26\sqrt{5}\ \dfrac{\text{feet}^2}{\text{second}}$

(b) $x(5)x'(5) = \sqrt{130}\ \dfrac{\text{feet}^2}{\text{second}}$

(c) $x(5)x'(5) = \dfrac{50\sqrt{2}}{3}\ \dfrac{\text{feet}^2}{\text{second}}$

(d) $x(5)x'(5) = \dfrac{800}{3}\ \dfrac{\text{feet}^2}{\text{second}}$

(e) None of the above

28. Suppose we have ten different cards, labeled 1-10, and a game consists of turning over two of the ten cards. If the numbers on the cards sum to 9, you win \$10 and if the numbers on the cards sum to 10, you win \$6. Otherwise, you lose \$2. What is the average winnings rounded to the nearest cent?

    (a) $-\$0.22$
    (b) \$0.20
    (c) \$1.40
    (d) \$6.00
    (e) None of the above

29. Consider the following statement:

    Condition X is true $\Leftrightarrow$ For any $y > 0$, there exists a natural number N, such that $n > N$ implies $a_n > y$.

    What does it mean for Condition X to be false?

    (a) Condition X is false $\Leftrightarrow$ There exists $y > 0$, such that for any natural number N, we have $n > N$ implies $a_n \leq y$.
    (b) Condition X is false $\Leftrightarrow$ There exists $y > 0$, such that for any natural number N, there exists $n > N$ with $a_n \leq y$.
    (c) Condition X is false $\Leftrightarrow$ There exists $y > 0$, such that for any natural number N, we have $n \leq N$ implies $a_n \leq y$.
    (d) Condition X is false $\Leftrightarrow$ There exists $y > 0$, such that for any natural number N, there exists $n \leq N$ with $a_n \leq y$.
    (e) None of the above

30. Suppose a is a real number and define

$$f(x) = \begin{cases} x^a & \text{if } x > 0 \\ 0 & \text{if } x \leq 0. \end{cases}$$

    What is the smallest value of a for which $f'(x)$ exists on $(-\infty, \infty)$?

    (a) $a = 0$
    (b) $a = \dfrac{1}{2}$
    (c) $a = 1$
    (d) $a = \dfrac{3}{2}$
    (e) None of the above