

# Monopoly Power with Correlated Markets

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*This paper considers the optimal output decision of duopolistic firms when markets are stochastic correlated. Its purpose is to show the intrinsic effect of the risk correlation under diversification. Particularly, impacts on supply, price level and hence the degree of market power are examined. The results suggest consumers do not always benefit from diversification. Such findings have strong anti-trust policy implications given the globalization of trade.*

There are many explanations for diversifications.<sup>1</sup> The main motive is usually to lower risk. Unfortunately, not all diversifications result in greater certainty. The impact of diversification on cross-market operations is ambiguous because of some shocks being dampened by diversifying while others are amplified. The contributing factor is the market correlation that we do observe in reality.<sup>2</sup> Our objective is to highlight conditions under which diversifications benefit the economy. Furthermore, concerns arise about the appropriateness of government regulations.

Obviously, diversifications that lead to greater supply and lower price are desirable. However, diversifications are marred by the enhanced market power that hurts consumers. There are numerous papers examining such an effect. Edwards (1955) investigates how the monopoly power can be sustained in a multi-market framework. Bulow et al. (1985) extend the research looking at the strategic interactions and find conglomerate power among oligopolies. Encaoua et al. (1986) consider the cross-elasticity of demand for goods confirming diversifications involve price manipulations. Rey & Tirole (1986) admit the disastrous anti-competitive power through diversification from the resale price maintenance and exclusive territories practices. Bernheim &

Whinston (1990) even show tacit collusion can result from diversification. Hughes & Oughton (1993) elaborate the work of Encaoua et al. by emphasizing the repeated contracts and once again support the potential of monopoly power. This paper brings up the uncertainty to these models and specifically emphasizes the intrinsic effect of risk correlation. Interestingly, the analysis is thin except a few papers from Eckel & Smith (1993), and Anam, Chiang & Shrestha (1996).

When firms decide to operate in the same market, competition can be intensified. Whether or not diversification results in lower prices is still controversial. While consumers prefer diversification when prices are lower, the competitive effect discourages firms to diversify. It follows that the interests of consumers may not well align with the producer interests. The analysis of such a conflict as seen by the price-cost margin forms the core of this paper.

As we explicitly incorporate related shocks in the demand, the risk correlation throws new light on diversification. Depending on whether markets are positively or negatively correlated, diversification can generate different results. For independent products, diversification associated with negative correlation amplifies the across-market competition, subsequently, raising the industry output and decreasing the mean

price. In this case, diversification is desirable for consumers. Unfortunately, the positive correlation gives a counter case to the contention that diversification always benefits consumers.

Cross-elasticity of demand deserves some attention. For the case of substitutes, Coke and Pepsi for example, the positive covariance (which reduces the equilibrium industry supply and increases the mean price) favors producers

from diversification. Consequently, consumers suffer from a higher degree of monopoly. With complementary products such as tire and rim, a negative market correlation reinforces the competitive effect of diversification and consumers are therefore better off. Such findings have strong policy implications on multi-market operations under uncertainty, particularly with regulatory concerns. Diversification and monopolis-

tic behavior are not a forgone conclusion. So far, the role of market covariance has been undermined for testing market competitiveness. Mergers and acquisitions resulting from diversification should be subject to a close examination of the underlying stochastic elements. Section 2 outlines the model and the main results. Section 3 is devoted to concluding comments.

## 2. THE MODEL

Consider a simple economy with two markets (1 and 2) subject to random and correlated shocks. These disturbances hit demands in an additive fashion. Accordingly, it follows that:

$$P_1 = a - Q_1 - \beta Q_2 + \mu_1$$

$$P_2 = a - \beta Q_1 - Q_2 + \mu_2$$

where  $-1 \leq \beta \leq 1$  with  $\beta > 0$  for substitute goods,  $\beta < 0$  for complement goods; and  $\mu_1$  and  $\mu_2$  are random variables with  $E\mu_1 = E\mu_2 = 0$ ,  $E(\mu_1^2) = E(\mu_2^2) = \sigma^2$ ,  $Cov(\mu_1, \mu_2) = E[(\mu_1 - E\mu_1)(\mu_2 - E\mu_2)] = E\mu_1\mu_2 = \sigma_{12} \neq 0$ .

Two possible cases of duopoly are considered. First, under the diversifying case, both firms (s and t) operate in both markets (1 and 2). Second, under the non-diversifying case, there exist four firms: s and t in market 1; h and k in market 2. For notational convenience, markets are referred by subscripts (1 or 2) and firms are denoted by superscripts (s, t, h or k). Total output for market 1 and 2 are given by:

$$Q_1 = \begin{cases} q_1^s + q_1^t & \text{case(I)} \\ q_1^s + q_1^t & \text{case(II)} \end{cases} \quad Q_2 = \begin{cases} q_2^s + q_2^t & \text{case(I)} \\ q_2^h + q_2^k & \text{case(II)} \end{cases}$$

Firms are assumed to have the same constant unit costs,  $c$  where  $c < a$ .

Suppose the expected utility of profit<sup>3</sup> for each firm is given by

$EU(\pi^i) = E(\pi^i) - 0.5R_A^i \text{Var}(\pi^i)$  with  $I = s, t, h$  and  $k$ , where  $U(\pi^i)$  is the mean-variance utility function and  $R_A^i \geq 0$  is the absolute risk-aversion coefficient. For simplicity of exposition, let firms be symmetric in all respects including risk attitudes. Assuming expected utility maximizing behavior and zero conjectural variations, the following results can be derived.

### Case (I): Diversification

Equilibrium output ( $q_D^*$ ) of each firm is

$$q_1^s = q_2^s = q_1^t = q_2^t = q_D^* = \frac{a + \mu - c}{3(1 + \beta) + R_A(\sigma^2 + \sigma_{12})}$$

Industry output ( $Q_D^*$ ) becomes

$$Q_1 = Q_2 = Q_D^* = \frac{2(a + \mu - c)}{3(1 + \beta) + R_A(\sigma^2 + \sigma_{12})}$$

(See appendix A for details)

For simplicity, further assume  $a=1$  and  $\mu=0$ . The corresponding mean price ( $EP_D$ ) in each market can be derived with two steps. First, substitute the industry output back into the demand equation and second apply the expectation over the entire expression. Due to symmetry, we have:

$$EP_1 = EP_2 = EP_D = \frac{1 + \beta + 2c(1 + \beta) + R_A(\sigma^2 + \sigma_{12})}{3(1 + \beta) + R_A(\sigma^2 + \sigma_{12})}$$

According to the Lerner index, the gap between price and marginal cost represents how much market power the firm has. The larger the gap, the higher the degree of monopoly power. In this diversifying case, the (expected) price-cost margin ( $L_D$ ) is given by:

$$L_D = \frac{EP_D - c}{EP_D} = \frac{1 + \beta - c[1 + \beta + R_A(\sigma^2 + \sigma_{12})] + R_A(\sigma^2 + \sigma_{12})}{1 + \beta + 2c(1 + \beta) + R_A(\sigma^2 + \sigma_{12})}$$

## Case (II): Non-diversification

As each firm produces the same output ( $q_N^*$ ) by symmetry, equilibrium solution yields:

$$q_1^s = q_1^t = q_2^h = q_2^k = q_N^* = \frac{a + \mu - c}{3 + 2\beta + R_A \sigma^2}$$

The total output ( $Q_N^*$ ) in each market is:

$$Q_1 = Q_2 = Q_N^* = \frac{2(a + \mu - c)}{3 + 2\beta + R_A \sigma^2}$$

(See appendix B for details)

Following the same procedure as before and also assuming  $a = 1$  and  $\mu = 0$ , we have the mean price as below.

$$EP_1 = EP_2 = EP_N = \frac{1 + 2c(1 + \beta) + R_A \sigma^2}{3 + 2\beta + R_A \sigma^2}$$

Then, the expected price-cost margin under non-diversification becomes:

$$L_N = \frac{EP_N - c}{EP_N} = \frac{(1 - c)(1 + R_A \sigma^2)}{1 + 2c(1 + \beta) + R_A \sigma^2}$$

Now, compare the optimal output levels under diversification and non-diversification.

$$q_D - q_N = \frac{a + \mu - c}{3(1 + \beta) + R_A(\sigma^2 + \sigma_{12})} - \frac{a + \mu - c}{3 + 2\beta + R_A \sigma^2} <(>) 0 \text{ if } \beta + R_A \sigma_{12} >(<) 0$$

Clearly, both the covariance and the substitutability/complementary drive the optimal output. Given the fact that  $-1 \leq \beta \leq 1$  and  $-\sigma^2 \leq \sigma_{12} \leq \sigma^2$ , the following statements can be made for the case of  $R_A > 0$ .

(1) If  $\beta > 0$  and  $\sigma_{12} > 0$ , then  $q_D < q_N$ ,  $EP_D > EP_N$  and  $L_D > L_N$ .

This implies with substitute goods, diversification leads to a decreased supply when markets are positively correlated. However, profit margin is higher under diversification. The price of the diversified good is higher than if the oligopoly firm only produces one specialized good.

(2) If  $\beta = 0$  and  $\sigma_{12} > 0$ , then  $q_D < q_N$ ,  $EP_D > EP_N$  and  $L_D > L_N$ .

(3) If  $\beta = 0$  and  $\sigma_{12} < 0$ , then  $q_D > q_N$ ,  $EP_D < EP_N$  and  $L_D < L_N$ .

This implies, in the case of independent goods, diversification may lower or raise firm supply depending precisely on the positive or negative market correlation. If markets are positively correlated, both expected profit margin and price are higher under diversification. If markets are negatively correlated, then the opposite is the case.

(4) If  $\beta > 0$  and  $\sigma_{12} = 0$ , then  $q_D < q_N$ ,  $EP_D > EP_N$  and  $L_D > L_N$ .

(5) If  $\beta < 0$  and  $\sigma_{12} = 0$ , then  $q_D > q_N$ ,  $EP_D < EP_N$  and  $L_D < L_N$ .

This implies the substitutability and complementary status of goods becomes the critical factors when market demands are stochastically independent. This is exactly the same result suggested by Hughes & Oughton (1993). On average, prices of diversified goods are higher than those of non-diversified oligopoly firms when goods are substitutes. The opposite case results when goods are complementary. Further, the expected profit margin is directly related to the status of the goods. In this case, substitute goods result in higher expected profit margins for firms that diversify, whereas complementary goods result in higher profit margins for non-diversified firms.<sup>4</sup>

(6) If  $\beta < 0$  and  $\sigma_{12} < 0$ , then  $q_D > q_N$ ,  $EP_D < EP_N$  and  $L_D < L_N$ .

This implies diversification results in an increase of output when goods are complementary and markets are negatively correlated.

Average prices of diversified goods are lower than those offered by non-diversifying firms. In this case, profit margins for firms that engage in diversification are lower than those that remain active in only one market. Further, when markets are perfectly negatively correlated, firms will produce unlimited amount of diversified complementary goods.<sup>5</sup> This result resembles perfect competition.

(7a) If  $\beta > 0$  and  $\sigma_{12} < 0$  and  $|\beta/R_A| > |\sigma_{12}|$ , then  $q_D < q_N$ ,  $EP_D > EP_N$  and  $L_D > L_N$ .

(7b) If  $\beta > 0$  and  $\sigma_{12} > 0$  and  $|\beta/R_A| < |\sigma_{12}|$ , then  $q_D > q_N$ ,  $EP_D < EP_N$  and  $L_D < L_N$ .

This implies the magnitude of the substitutability status of the good is critical in determining the supply, expected price and profit margin. Given that markets are negatively correlated and goods are substitutes, diversification leads to a smaller supply when the absolute value of  $\beta/R_A$  is greater than the absolute value of the covariance between markets. The opposite is the case when the absolute value of covariance is greater than the absolute value of  $\beta/R_A$ .

(8a) If  $\beta < 0$  and  $\sigma_{12} > 0$  and  $|\beta/R_A| > |\sigma_{12}|$ , then  $q_D > q_N$ ,  $EP_D < EP_N$  and  $L_D < L_N$ .

(8b) If  $\beta < 0$  and  $\sigma_{12} > 0$  and  $|\beta/R_A| < |\sigma_{12}|$ , then  $q_D < q_N$ ,  $EP_D > EP_N$  and  $L_D > L_N$ .

In determining the supply, expected price and profit margin, the magnitude of the complementary status of the commodity is critical. When markets are positively correlated, with complementary goods, diversification leads to a larger supply when the absolute value of  $\beta/R_A$  is greater than the absolute value of the covariance between markets.<sup>6</sup> The opposite is the case when the absolute value of the market covariance is greater than the absolute value of  $\beta/R_A$ . Indeed, using some simulation parameters, we can visualize the different effects of  $\beta$  and  $\sigma_{12}$  with 3-dimensional figures.<sup>7</sup>

In the above discussion, the random variables of  $\mu_1$  and  $\mu_2$  are assumed to be equal. Although they represent the price variability of the goods, the variance of profit for the firm is represented by  $\sigma^2$ . While firms maximize expected utility, they also pay attention to the expected profit margins upon which their diversification decisions are based. From the firms' perspectives, there are circumstances, especially with a smaller Lerner index, choosing not to diversify to avoid head-to-head competition makes perfect economic sense. As a result, consumers suffer from a higher degree of monopoly. Then, government intervention may be needed. Under some other conditions associated with larger Lerner indices, firms have incentive to diversify basically for their own self-interests. Surprisingly, consumers also benefit. When this occurs, anti-monopolistic policies are redundant.

### 3. Conclusions

This paper follows closely with the existing models except that market demands are stochastically correlated. We show not only the demand elasticity, but also the risk correlation can form the basis for exercising market power. Since the sign and magnitude of those market parameters drive the individual optimal output level, diversification may generate different results. Under some circumstances, diversification can benefit producers without benefiting consumers, and vice versa. In some situations, the producer is indifferent while consumers benefit with increased supply. In some conditions, especially when goods and markets are independent, both the firm and consumers are indifferent to the decision to diversify. It is demonstrated that negative covariance yields a competitive outcome ( $L_D < L_N$ ) whenever the demands of two products are independent except for their stochastic components. The intuition is that negative demand correlation raises the industry output and decreases the average price. The consumers gain in this situation. The same conclusion can also apply for complementary products. However, a positive covariance coupled with substitute goods tends to produce a non-competitive outcome via diversification. All of these suggest stochastic

correlation between markets can increase or decrease the expected supernormal profits and hence the degree of monopoly power. Given the globalization of trade, positive correlation in demand among markets is indeed very possible. Therefore, the stochastic correlation between markets becomes a critical factor to examine. This theoretical paper is unlikely to be empirically irrelevant, even with the difficulties associated with quantifying the status of goods. Its central results have strong implications for the design of any competition policy. Firms may naturally be inclined to diversify, entering into competitive markets in order to increase profit margins, thereby resulting in anti-monopolistic initiatives. Analysis of asymmetric demand and utility functions would be an extension for future research.

### Footnotes

- 1 We use the term diversification to include horizontal or vertical mergers.
- 2 It is a real conjecture that success in North American markets has stimulated demand for Hard Rock Cafes in China. This is an example of positive marketing correlation.
- 3 Epstein (1985) has shown with the non-expected utility approach the concept of decreasing risk aversion is equivalent to the mean-variance formulation. Particularly, the linear form used in this text is consistent with a constant degree of risk aversion.
- 4 If  $\beta=0$  &  $\sigma_{12}=0$ , then  $q_D=q_N$ ,  $EP_D=EP_N$  and  $L_D=L_N$ .
- 5 If  $\beta=-1$  &  $\sigma_{12}=-\sigma^2$ , then  $q_D=\infty$ ,  $EP_D=0$  and  $L_D=1$
- 6 Similar competitive results are obtained as the polar case (6a).
- 7 Due to space limitation, the graphs are not provided. However, a completed graphical analysis will be available from the author upon request.

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**APPENDIX A**  
**Case I (Diversification)**

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The expected utility of profit for firm  $s$  is:

$$EU(\pi_D^s) = (a - Q_1 - \beta Q_2 + \underline{\mu} - c)q_1^s + (a - \beta Q_1 - Q_2 + \underline{\mu} - c)q_2^s - 0.5R_A[(q_1^s)^2\sigma^2 + (q_2^s)^2\sigma^2 + 2q_1q_2\sigma_{12}]$$

The first-order conditions are:

$$\frac{\partial EU(\pi_D^s)}{\partial q_1^s} = a - \beta(2q_2^s + q_2^t) - 2q_1^s - q_1^t - R_A(q_1^s\sigma^2 + q_2^s\sigma_{12}) + \underline{\mu} - c = 0$$

$$\frac{\partial EU(\pi_D^s)}{\partial q_2^s} = a - \beta(2q_1^s + q_1^t) - 2q_2^s - q_2^t - R_A(q_2^s\sigma^2 + q_1^s\sigma_{12}) + \underline{\mu} - c = 0$$

Given the symmetry of firms, i.e.,  $q_1^s = q_2^t$  and  $q_2^s = q_1^t$ , equilibrium output of each firm under diversification is:

$$q_1^s = q_2^s = q_1^t = q_2^t = q_D^* = \frac{a + \underline{\mu} - c}{3(1 + \beta) + R_A(\sigma^2 + \sigma_{12})}$$

$$\text{Industry output is } Q_1 = Q_2 = 2q_D^* = Q_D^* = \frac{2(a + \underline{\mu} - c)}{3(1 + \beta) + R_A(\sigma^2 + \sigma_{12})} \text{ Q.E.D.}$$


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**APPENDIX B**  
**Case II (Non-diversification)**

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The expected utility of profit for firm  $s$  is:

$$EU(\pi_N^s) = (a - Q_1 - \beta Q_2 + \underline{\mu} - c)q_1^s - 0.5R_A(q_1^s)^2\sigma^2$$

The first-order condition is:

$$\frac{\partial EU(\pi_N^s)}{\partial q_1^s} = a - \beta(q_2^h + q_2^k) - 2q_1^s - q_1^t - R_A(q_1^s\sigma^2) + \underline{\mu} - c = 0$$

The symmetry assumption requires each firm producing the same output, i.e.,  $q_1^s = q_1^t = q_2^h = q_2^k$ , equilibrium output under non-diversification is:

$$q_1^s = q_1^t = q_2^h = q_2^k = q_N^* = \frac{a + \underline{\mu} - c}{3 + 2\beta + R_A\sigma^2}$$

$$\text{Industry output is } Q_1 = Q_2 = 2q_N^* = Q_N^* = \frac{2(a + \underline{\mu} - c)}{3 + 2\beta + R_A\sigma^2} \text{ Q.E.D.}$$


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